Mathematica 11.3 Integration Test Results

Test results for the 166 problems in "7.2.2 (d x)^m (a+b arccosh(c x))^n.m"

Problem 39: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCosh}[a\,x]^4}{x^2}\,dx$$

Optimal (type 4, 150 leaves, 11 steps):

$$-\frac{\text{ArcCosh}[a\,x]^4}{x}+8\,a\,\text{ArcCosh}[a\,x]^3\,\text{ArcTan}\left[e^{\text{ArcCosh}[a\,x]}\right]-12\,i\,a\,\text{ArcCosh}[a\,x]^2\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[a\,x]}\right]+12\,i\,a\,\text{ArcCosh}[a\,x]^2\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[a\,x]}\right]+24\,i\,a\,\text{ArcCosh}[a\,x]\,\text{PolyLog}\left[3,-i\,e^{\text{ArcCosh}[a\,x]}\right]-24\,i\,a\,\text{ArcCosh}[a\,x]\,\text{PolyLog}\left[3,i\,e^{\text{ArcCosh}[a\,x]}\right]-24\,i\,a\,\text{PolyLog}\left[4,-i\,e^{\text{ArcCosh}[a\,x]}\right]+24\,i\,a\,\text{PolyLog}\left[4,i\,e^{\text{ArcCosh}[a\,x]}\right]$$

Result (type 4, 478 leaves):

$$a\left(-\frac{7\,i\,\pi^4}{16}+\frac{1}{2}\pi^3\,\text{ArcCosh}[a\,x]-\frac{3}{2}\,i\,\pi^2\,\text{ArcCosh}[a\,x]^2-2\,\pi\,\text{ArcCosh}[a\,x]^3+i\,\text{ArcCosh}[a\,x]^4-\frac{\text{ArcCosh}[a\,x]^4}{a\,x}+\frac{1}{2}\pi^3\,\text{Log}\left[1+i\,e^{-\text{ArcCosh}[a\,x]}\right]-3\,i\,\pi^2\,\text{ArcCosh}[a\,x]\,\text{Log}\left[1+i\,e^{-\text{ArcCosh}[a\,x]}\right]-6\,\pi\,\text{ArcCosh}[a\,x]^2\,\text{Log}\left[1+i\,e^{-\text{ArcCosh}[a\,x]}\right]+4\,i\,\text{ArcCosh}[a\,x]^3\,\text{Log}\left[1+i\,e^{-\text{ArcCosh}[a\,x]}\right]+3\,i\,\pi^2\,\text{ArcCosh}[a\,x]\,\text{Log}\left[1-i\,e^{\text{ArcCosh}[a\,x]}\right]+6\,\pi\,\text{ArcCosh}[a\,x]^2\,\text{Log}\left[1-i\,e^{\text{ArcCosh}[a\,x]}\right]-\frac{1}{2}\pi^3\,\text{Log}\left[1+i\,e^{\text{ArcCosh}[a\,x]}\right]-4\,i\,\text{ArcCosh}[a\,x]^3\,\text{Log}\left[1+i\,e^{\text{ArcCosh}[a\,x]}\right]+\frac{1}{2}\pi^3\,\text{Log}\left[\text{Tan}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCosh}[a\,x]\right)\right]\right]+3\,i\left(\pi-2\,i\,\text{ArcCosh}[a\,x]\right)^2\,\text{PolyLog}\left[2,-i\,e^{-\text{ArcCosh}[a\,x]}\right]-12\,i\,\text{ArcCosh}[a\,x]^2\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[a\,x]}\right]+3\,i\,\pi^2\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[a\,x]}\right]+12\,\pi\,\text{ArcCosh}[a\,x]\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[a\,x]}\right]+12\,\pi\,\text{PolyLog}\left[3,-i\,e^{-\text{ArcCosh}[a\,x]}\right]-24\,i\,\text{ArcCosh}[a\,x]\,\text{PolyLog}\left[3,-i\,e^{-\text{ArcCosh}[a\,x]}\right]+24\,i\,\text{ArcCosh}[a\,x]\,\text{PolyLog}\left[3,-i\,e^{\text{ArcCosh}[a\,x]}\right]-12\,\pi\,\text{PolyLog}\left[3,i\,e^{\text{ArcCosh}[a\,x]}\right]-24\,i\,\text{PolyLog}\left[4,-i\,e^{-\text{ArcCosh}[a\,x]}\right]-24\,i\,\text{PolyLog}\left[4,-i\,e^{\text{ArcCosh}[a\,x]}\right]\right)$$

Problem 41: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCosh}[a\,x]^4}{x^4}\,dx$$

Optimal (type 4, 268 leaves, 19 steps) :

$$\frac{2 a^2 \text{ArcCosh}[a x]^2}{x}+\frac{2 a \sqrt{-1+a x} \sqrt{1+a x} \text{ArcCosh}[a x]^3}{3 x^2}-\frac{\text{ArcCosh}[a x]^4}{3 x^3}-8 a^3 \text{ArcCosh}[a x] \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right]+\frac{4}{3} a^3 \text{ArcCosh}[a x]^3 \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right]+4 i a^3 \text{PolyLog}\left[2,-i e^{\text{ArcCosh}[a x]}\right]-2 i a^3 \text{ArcCosh}[a x]^2 \text{PolyLog}\left[2,-i e^{\text{ArcCosh}[a x]}\right]-4 i a^3 \text{PolyLog}\left[2, i e^{\text{ArcCosh}[a x]}\right]+2 i a^3 \text{ArcCosh}[a x]^2 \text{PolyLog}\left[2, i e^{\text{ArcCosh}[a x]}\right]+4 i a^3 \text{ArcCosh}[a x] \text{PolyLog}\left[3,-i e^{\text{ArcCosh}[a x]}\right]-4 i a^3 \text{ArcCosh}[a x] \text{PolyLog}\left[3, i e^{\text{ArcCosh}[a x]}\right]-4 i a^3 \text{PolyLog}\left[4,-i e^{\text{ArcCosh}[a x]}\right]+4 i a^3 \text{PolyLog}\left[4, i e^{\text{ArcCosh}[a x]}\right]$$

Result (type 4, 595 leaves) :

$$a^3 \left(\frac{1}{2} i \left(8+\pi^2-4 i \pi \text{ArcCosh}[a x]-4 \text{ArcCosh}[a x]^2\right) \text{PolyLog}\left[2,-i e^{-\text{ArcCosh}[a x]}\right]-\frac{1}{96} i \left(7 \pi^4+8 i \pi^3 \text{ArcCosh}[a x]+24 \pi^2 \text{ArcCosh}[a x]^2+\frac{192 i \text{ArcCosh}[a x]^2}{a x}-32 i \pi \text{ArcCosh}[a x]^3+\frac{64 i \sqrt{\frac{-1+a x}{1+a x}} (1+a x) \text{ArcCosh}[a x]^3}{a^2 x^2}-16 \text{ArcCosh}[a x]^4-\frac{32 i \text{ArcCosh}[a x]^4}{a^3 x^3}-384 \text{ArcCosh}[a x] \text{Log}\left[1-i e^{-\text{ArcCosh}[a x]}\right]+8 i \pi^3 \text{Log}\left[1+i e^{-\text{ArcCosh}[a x]}\right]+384 \text{ArcCosh}[a x] \text{Log}\left[1+i e^{-\text{ArcCosh}[a x]}\right]+48 \pi^2 \text{ArcCosh}[a x] \text{Log}\left[1+i e^{-\text{ArcCosh}[a x]}\right]-96 i \pi \text{ArcCosh}[a x]^2 \text{Log}\left[1+i e^{-\text{ArcCosh}[a x]}\right]-64 \text{ArcCosh}[a x]^3 \text{Log}\left[1+i e^{-\text{ArcCosh}[a x]}\right]-48 \pi^2 \text{ArcCosh}[a x] \text{Log}\left[1-i e^{\text{ArcCosh}[a x]}\right]+96 i \pi \text{ArcCosh}[a x]^2 \text{Log}\left[1-i e^{\text{ArcCosh}[a x]}\right]-8 i \pi^3 \text{Log}\left[1+i e^{\text{ArcCosh}[a x]}\right]+64 \text{ArcCosh}[a x]^3 \text{Log}\left[1+i e^{\text{ArcCosh}[a x]}\right]+8 i \pi^3 \text{Log}\left[\text{Tan}\left[\frac{1}{4} (\pi+2 i \text{ArcCosh}[a x])\right]\right]+384 \text{PolyLog}\left[2, i e^{-\text{ArcCosh}[a x]}\right]+192 \text{ArcCosh}[a x]^2 \text{PolyLog}\left[2,-i e^{\text{ArcCosh}[a x]}\right]-48 \pi^2 \text{PolyLog}\left[2, i e^{\text{ArcCosh}[a x]}\right]+192 i \pi \text{ArcCosh}[a x] \text{PolyLog}\left[2, i e^{\text{ArcCosh}[a x]}\right]+192 i \pi \text{PolyLog}\left[3,-i e^{-\text{ArcCosh}[a x]}\right]+384 \text{ArcCosh}[a x] \text{PolyLog}\left[3,-i e^{-\text{ArcCosh}[a x]}\right]-384 \text{ArcCosh}[a x] \text{PolyLog}\left[3,-i e^{\text{ArcCosh}[a x]}\right]-192 i \pi \text{PolyLog}\left[3, i e^{\text{ArcCosh}[a x]}\right]+384 \text{PolyLog}\left[4,-i e^{-\text{ArcCosh}[a x]}\right]+384 \text{PolyLog}\left[4,-i e^{\text{ArcCosh}[a x]}\right]\right)\right)$$

Problem 117: Unable to integrate problem.

$$\int x^m \text{ArcCosh}[a x]^2 \, dx$$

Optimal (type 5, 167 leaves, 2 steps):

$$\frac{x^{1+m} \text{ArcCosh}[a x]^2}{1+m} - \left(2 a x^{2+m} \sqrt{1-a^2 x^2} \text{ ArcCosh}[a x] \text{ Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]\right) \Big/ \left(\left(2+3 m+m^2\right) \sqrt{-1+a x} \sqrt{1+a x}\right) - \left(2 a^2 x^{3+m} \text{ HypergeometricPFQ}\left[\left\{1, \frac{3}{2}+\frac{m}{2}, \frac{3}{2}+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}, \frac{5}{2}+\frac{m}{2}\right\}, a^2 x^2\right]\right) \Big/ \left(6+11 m+6 m^2+m^3\right)$$

Result (type 8, 12 leaves):

$$\int x^m \text{ArcCosh}[a x]^2 \, dx$$

Problem 118: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^m \text{ArcCosh}[a x] \, dx$$

Optimal (type 5, 91 leaves, 4 steps):

$$\frac{x^{1+m} \text{ArcCosh}[a x]}{1+m} - \frac{a x^{2+m} \sqrt{1-a^2 x^2} \text{ Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{\left(2+3 m+m^2\right) \sqrt{-1+a x} \sqrt{1+a x}}$$

Result (type 6, 329 leaves):

$$\frac{1}{1+m} x^m \left(-\left(\left(12 \sqrt{-1+a x} \sqrt{1+a x} \text{ AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-a x, \frac{1}{2} (1-a x)\right]\right) \Big/ \left(a \left(6 \text{ AppellF1}\left[\frac{1}{2}, -m, -\frac{1}{2}, \frac{3}{2}, 1-a x, \frac{1}{2} (1-a x)\right] + (-1+a x) \left(4 m \text{ AppellF1}\left[\frac{3}{2}, 1-m, -\frac{1}{2}, \frac{5}{2}, 1-a x, \frac{1}{2} (1-a x)\right] + \text{AppellF1}\left[\frac{3}{2}, -m, \frac{1}{2}, \frac{5}{2}, 1-a x, \frac{1}{2} (1-a x)\right]\right)\right)\right)\right) + \left(12 \sqrt{\frac{-1+a x}{1+a x}} \text{ AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-a x, \frac{1}{2} (1-a x)\right]\right) \Big/ \left(a \left(6 \text{ AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 1-a x, \frac{1}{2} (1-a x)\right] + (-1+a x) \left(4 m \text{ AppellF1}\left[\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 1-a x, \frac{1}{2} (1-a x)\right] - \text{AppellF1}\left[\frac{3}{2}, -m, \frac{3}{2}, \frac{5}{2}, 1-a x, \frac{1}{2} (1-a x)\right]\right)\right)\right) + x \text{ ArcCosh}[a x]\right)$$

Problem 163: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{f x}\,(a+b\,\text{ArcCosh}[c x])^2\,dx$$

Optimal (type 5, 141 leaves, 2 steps):

$$\frac{2 (f x)^{3/2} (a+b\,\text{ArcCosh}[c x])^2}{3 f}-\left(8 b c (f x)^{5/2}\sqrt{1-c^2 x^2}\,(a+b\,\text{ArcCosh}[c x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2 x^2\right]\right)\Big/\left(15 f^2\sqrt{-1+c x}\,\sqrt{1+c x}\right)-\frac{16 b^2 c^2 (f x)^{7/2}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2 x^2\right]}{105 f^3}$$

Result (type 5, 256 leaves):

$$\frac{1}{27}\sqrt{f x}\left(18 a^2 x+36 a b x\,\text{ArcCosh}[c x]-\frac{24 b^2\sqrt{\frac{-1+c x}{1+c x}}\,(1+c x)\,\text{ArcCosh}[c x]}{c}+2 b^2 x\left(8+9\,\text{ArcCosh}[c x]^2\right)-\frac{24 a b\left(\sqrt{-1+c x}\,(1+c x)+\frac{i\sqrt{\frac{1+c x}{-1+c x}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{1}{\sqrt{-1+c x}}\right],2\right]}{\sqrt{\frac{c x}{-1+c x}}}\right)}{c\sqrt{1+c x}}+\frac{1}{c}24 b^2\sqrt{\frac{-1+c x}{1+c x}}\,(1+c x)\,\text{ArcCosh}[c x]\,\text{Hypergeometric2F1}\left[\frac{3}{4},1,\frac{5}{4},c^2 x^2\right]-\frac{3\sqrt{2}\,b^2\pi x\,\text{HypergeometricPFQ}\left[\left\{\frac{3}{4},\frac{3}{4},1\right\},\left\{\frac{5}{4},\frac{7}{4}\right\},c^2 x^2\right]}{\text{Gamma}\left[\frac{5}{4}\right]\text{Gamma}\left[\frac{7}{4}\right]}\right)$$

Problem 164: Unable to integrate problem.

$$\int (d x)^m\,(a+b\,\text{ArcCosh}[c x])^2\,dx$$

Optimal (type 5, 194 leaves, 2 steps):

$$\frac{(d\,x)^{1+m}\,(a+b\,\text{ArcCosh}[c\,x])^2}{d\,(1+m)}-\left(2\,b\,c\,(d\,x)^{2+m}\,\sqrt{1-c^2\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2\,x^2\right]\right)\Big/\left(d^2\,(1+m)\,(2+m)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-\left(2\,b^2\,c^2\,(d\,x)^{3+m}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2}\right\},\left\{2+\frac{m}{2},\frac{5}{2}+\frac{m}{2}\right\},c^2\,x^2\right]\right)\Big/\left(d^3\,(1+m)\,(2+m)\,(3+m)\right)$$

Result (type 8, 18 leaves) :

$$\int (d\,x)^m\,(a+b\,\text{ArcCosh}[c\,x])^2\,dx$$

Problem 165: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (d\,x)^m\,(a+b\,\text{ArcCosh}[c\,x])\,dx$$

Optimal (type 5, 106 leaves, 4 steps) :

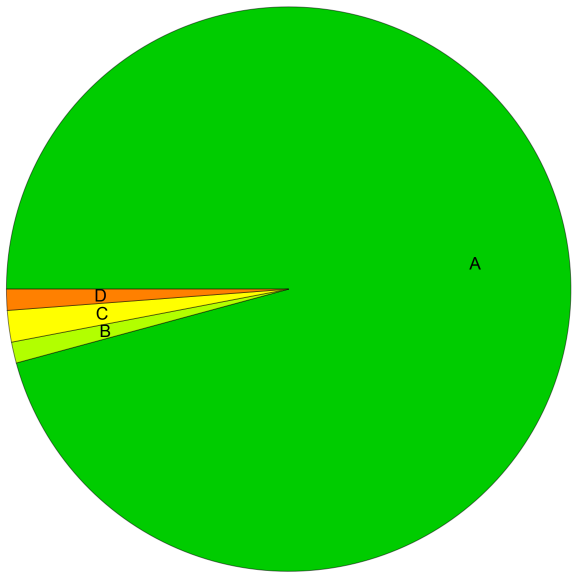
$$\frac{(d\,x)^{1+m}\,(a+b\,\text{ArcCosh}[c\,x])}{d\,(1+m)}-\frac{b\,c\,(d\,x)^{2+m}\,\sqrt{1-c^2\,x^2}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2\,x^2\right]}{d^2\,(1+m)\,(2+m)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 6, 337 leaves) :

$$\frac{1}{1+m}(d\,x)^m\Bigg(-\Bigg(\left(12\,b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{AppellF1}\left[\frac{1}{2},-m,-\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\Big/\Bigg(c\,\Bigg(6\,\text{AppellF1}\left[\frac{1}{2},-m,-\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+(-1+c\,x)\,\Bigg(4\,m\,\text{AppellF1}\left[\frac{3}{2},1-m,-\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+\text{AppellF1}\left[\frac{3}{2},-m,\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\Bigg)\Bigg)\Bigg)\Bigg)+\left(12\,b\,\sqrt{\frac{-1+c\,x}{1+c\,x}}\,\text{AppellF1}\left[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\right)\Big/\Bigg(c\,\Bigg(6\,\text{AppellF1}\left[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]+(-1+c\,x)\,\Bigg(4\,m\,\text{AppellF1}\left[\frac{3}{2},1-m,\frac{1}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]-\text{AppellF1}\left[\frac{3}{2},-m,\frac{3}{2},\frac{5}{2},1-c\,x,\frac{1}{2}\,(1-c\,x)\right]\Bigg)\Bigg)\Bigg)+x\,(a+b\,\text{ArcCosh}[c\,x])\Bigg)$$

Summary of Integration Test Results

166 integration problems



A - 159 optimal antiderivatives

B - 2 more than twice size of optimal antiderivatives

C - 3 unnecessarily complex antiderivatives

D - 2 unable to integrate problems

E - 0 integration timeouts